EJERCICIO 1 (1:45)

Calcular $U_{I\,(t,t')}$ sabiendo que cumple:

$$\boldsymbol{i\,\partial_t U_{I\,(t,t')} = H'_{I\,(t)} U_{I\,(t,t')}}$$

Integramos ambos términos entre t´ y t.

$$\int_{t'}^{t} i\,\partial_t U_{I\,(t_1,t')}\,dt_1 = i\big(U_{I\,(t,t')} - U_{I\,(t',t')}\big) = i\big(U_{I\,(t,t')} - 1\big)$$

Se ha considerado que $U_{I\,(t',\,t')}$ debe ser igual a 1 (el operador de evolución temporal entre un tiempo y otro igual a éste debe ser igual a 1).

$$i\big(U_{I\,(t,t')} - 1\big) = \int_{t'}^{t} dt_1\, H'_{I\,(t_1)} U_{I\,(t_1,t')}$$

[1] $\quad U_{I\,(t,t')} = 1 + (-i)\int_{t'}^{t} dt_1\, H'_{I\,(t_1)} U_{I\,(t_1,t')}$

$$U_{I\,(t_1,t')} = 1 + (-i)\int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)} U_{I\,(t_2,t')}$$

Reemplazando en [1]

$$U_{I\,(t,t')} = 1 + (-i)\int_{t'}^{t} dt_1\, H'_{I\,(t_1)}\left(1 + (-i)\int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)} U_{I\,(t_2,t')}\right)$$

[2] $\quad U_{I\,(t,t')} = 1 + (-i)\int_{t'}^{t} dt_1\, H'_{I\,(t_1)} + (-i)^2\int_{t'}^{t} dt_1\, H'_{I\,(t_1)}\left(\int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)} U_{I\,(t_2,t')}\right)$

$$U_{I\,(t_2,t')} = 1 + (-i)\int_{t'}^{t_2} dt_1\, H'_{I\,(t_1)} + (-i)^2\int_{t'}^{t_2} dt_1\, H'_{I\,(t_1)}\left(\int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)} U_{I\,(t_2,t')}\right)$$

Calculamos la última integral (cambiamos y subrayamos los últimos índices mudos).

$$\begin{aligned}
\int_{t'}^{t_1} dt_2\;\, H'_{I\,(t_2)} U_{I\,(t_2,t')}
&= \int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)}\left\{1 + (-i)\int_{t'}^{t_2} dt_1\, H'_{I\,(t_1)}\right.\\
&\left. + (-i)^2\int_{t'}^{t_2} dt_{\underline{3}}\, H'_{I\,(t_{\underline{3}})}\left(\int_{t'}^{t_{\underline{3}}} dt_{\underline{4}}\, H'_{I\,(t_{\underline{4}})} U_{I\,(t_{\underline{4}},t')}\right)\right\}\\
&= \int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)} + (-i)\int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)}\int_{t'}^{t_2} dt_1\, H'_{I\,(t_1)}\\
&+ (-i)^2\int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)}\int_{t'}^{t_2} dt_{\underline{3}}\, H'_{I\,(t_{\underline{3}})}\left(\int_{t'}^{t_{\underline{3}}} dt_{\underline{4}}\, H'_{I\,(t_{\underline{4}})} U_{I\,(t_{\underline{4}},t')}\right)
\end{aligned}$$

Reemplazando en [2]

$$U_{I\,(t,t')} = 1 + (-i)\int_{t'}^{t} H'_{I\,(t_1)}\; dt_1 + (-i)^2 \int_{t'}^{t} dt_1\, H'_{I\,(t_1)} \left\{ \int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)} + (-i) \int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)} \int_{t'}^{t_2} dt_3\, H'_{I\,(t_3)} + (-i)^2 \int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)} \int_{t'}^{t_2} dt_{\underline{3}}\, H'_{I\,(t_{\underline{3}})} \left(\int_{t'}^{t_{\underline{3}}} dt_{\underline{4}}\, H'_{I\,(t_{\underline{4}})} U_{I\,(t_{\underline{4}},t')} \right) \right\}$$

$$\boldsymbol{U_{I\,(t,t')} = 1 + (-i)\int_{t'}^{t} H'_{I\,(t_1)}\; dt_1 + (-i)^2 \int_{t'}^{t} dt_1\, H'_{I\,(t_1)} \int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)} + (-i)^3 \int_{t'}^{t} dt_1\, H'_{I\,(t_1)} \int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)} \int_{t'}^{t_2} dt_3\, H'_{I\,(t_3)} + (-i)^4 \int_{t'}^{t} dt_1\, H'_{I\,(t_1)} \int_{t'}^{t_1} dt_2\, H'_{I\,(t_2)} \int_{t'}^{t_2} dt_{\underline{3}}\, H'_{I\,(t_{\underline{3}})} \left(\int_{t'}^{t_{\underline{3}}} dt_{\underline{4}}\, H'_{I\,(t_{\underline{4}})} U_{I\,(t_{\underline{4}},t')} \right)} + \cdots$$

EJERCICIO 1 (28:49)

Calcular los vectores y valores propios del hamiltoniano del ejemplo "de juguete" de dimensión 2:

$$\boldsymbol{H} = \begin{pmatrix} \mathbf{100} & \mathbf{1} \\ \mathbf{1} & \mathbf{200} \end{pmatrix}$$

Calculamos los autovalores λ:

$$\det\begin{pmatrix} 100-\lambda & 1 \\ 1 & 200-\lambda \end{pmatrix} = 0$$

$$(100-\lambda)(200-\lambda) - 1 = 0$$

$$\boxed{\boldsymbol{\lambda_1 = E_0 = 150 - \sqrt{2501} = 99,990001}}$$

$$\boxed{\boldsymbol{\lambda_2 = E_1 = 150 + \sqrt{2501} = 200,009999}}$$

Autovector Ω

$$\begin{pmatrix} 100-\lambda_1 & 1 \\ 1 & 200-\lambda_1 \end{pmatrix}\begin{pmatrix} \Omega_1 \\ \Omega_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 100-\left(150-\sqrt{2501}\right) & 1 \\ 1 & 200-\left(150-\sqrt{2501}\right) \end{pmatrix}\begin{pmatrix} \Omega_1 \\ \Omega_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\sqrt{2501}-50\right)\Omega_1 + \Omega_2 = 0$$

Multiplicando la primera fila por $\left(\sqrt{2501}+50\right)$

$$\left(\sqrt{2501}-50\right)\left(\sqrt{2501}+50\right)\Omega_1 + \left(\sqrt{2501}+50\right)\Omega_2 = 0$$

$$(2501-2500)\Omega_1 + \left(\sqrt{2501}+50\right)\Omega_2 = 0$$

$$\Omega_2 = 1$$

$$\Omega_1 = -\left(\sqrt{2501}+50\right) = -100,0099990$$

Normalizando:

$$\boxed{|\boldsymbol{\Omega}\rangle = \begin{pmatrix} \mathbf{-0,9999500137} \\ \mathbf{9,99850\ \boldsymbol{x}\ 10^{-3}} \end{pmatrix}}$$

Autovector Ψ

$$\begin{pmatrix} 100-\lambda_2 & 1 \\ 1 & 200-\lambda_2 \end{pmatrix}\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 100-\left(150+\sqrt{2501}\right) & 1 \\ 1 & 200-\left(150+\sqrt{2501}\right) \end{pmatrix}\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(-\sqrt{2501}-50\right)\Psi_1 + \Psi_2 = 0$$

Multiplicando la primera fila por $\left(\sqrt{2501}-50\right)$

$$\left(-\sqrt{2501}-50\right)\left(\sqrt{2501}-50\right)\Psi_1+\left(\sqrt{2501}-50\right)\Psi_2=0$$

$$(-2501+2500)\Psi_1+\left(\sqrt{2501}-50\right)\Psi_2=0$$

$$\Psi_2=1$$

$$\Psi_1=\left(\sqrt{2501}-50\right)=0{,}0099990001$$

Normalizando:

$$\boxed{|\boldsymbol{\Psi}\rangle=\begin{pmatrix}\mathbf{9,998500387}\ \boldsymbol{x}\ \mathbf{10^{-3}}\\ \mathbf{0,9999500137}\end{pmatrix}}$$